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**SEMIPARAMETRIC ESTIMATION OF EFFORT FUNCTIONS  
WITH TWO LABOUR INPUTS: SOME EVIDENCE  
FROM FINNISH INDUSTRY DATA**

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ABSTRACT. Semi-parametric methods are introduced to estimate the effort function  $e(w)$  for unskilled and skilled workers using Finnish cross-sectional industry data from 1989. Well-defined effort function estimates are obtained only for unskilled workers. Non-linear labour demand models are also estimated semi-parametrically. Nonlinearities are identified using effort function estimates.

Keywords. Efficiency wage models, labour demand, semi-parametric estimation, IV-estimation.  
JEL classification: J41, J23, C14

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## 1. Introduction

Efficiency wage theories have attracted a lot of attention as a potential explanation for involuntary unemployment and other aspects of the labour market. However, testing the theories has been remarkably difficult. The common feature in many studies is that the basic element of efficiency wage models, the effort function,  $e(w)$ , is not estimated. Instead, different derived specifications that indirectly or directly incorporate the implications of efficiency wage theories have been the subject of empirical testing.

The approach of Krueger and Summers (1988) is to analyze observed inter-industry wage differentials. However, the evidence is at best indirect, because only the unexplained part of the differentials is identified with efficiency wage model predictions. Wadhvani and Wall (1991) and Konings and Walsh (1994) use a more direct approach. They show that worker productivity or some other performance index is positively related to the wages in a given firm relative to the utility the workers could get elsewhere. However finding is consistent with notions of compensating differentials and with bargaining theory (see also Machin and Manning 1992). Thus, the results cannot be interpreted as strongly in favour of efficiency wage theory (see also Kitazawa and Ohta 2002).

Ackum Agell (1994) carried out an interesting study using Swedish data, in which an estimate for effort was measured using time diary information. However, her results generally do not support the implications of efficiency wage theories (for similar results, see Fuess and Millea 2002). The approach advocated by Kumbhakar (1996) uses system modelling based on translog -production function, where the efficiency index of labour is identified with wage level and years of schooling. The results find evidence to support the view that wages paid by the farmers in rural India are efficiency wages. Goldsmith, Veun and Darity (2000) approach the efficiency wage hypothesis by emphasizing the aspects of employee motivation and personality aspects. They find with NLSY data that receiving an efficiency wage enhances a person's effort (see also Kugler 2003). These studies can be seen as promising alternatives to model and estimate the efficiency wage function directly. However, it is difficult to compare these results with our because of different methods and data used.

This paper advocates a general approach to estimate the effort function directly. The aim is to estimate the effort function as a part of the labour demand model using semi-parametric methods. This is done by estimating partially linear labour demand model augmented by a non-linear function of wages that gives an estimate for the effort function. Separate estimates are derived for both unskilled and skilled workers. Industry- and union-specific factors are controlled with dummy variables. Finnish cross-sectional industry data from 1989 is used. The results show that it is possible to identify a meaningful effort function only for unskilled workers. This is also supported by conducted specifications tests.

The structure of the paper is the following. The specifications to be estimated are derived from CES-type production function in Section 2. The CES approach gives analytically simple and identifiable solution between labour demand and effort function. Section 3. gives an introduction to semi-parametric IV methods used: smoothed splines and series estimators in addition to specifications tests. The results are presented in Section 4., and Section 5. is a short summary of the paper. The appendices give a more detailed description of some of the technical issues and data used.

## 2. The efficiency wage model

The literature suggests many different approaches for efficiency wage modelling. The general starting point in all efficiency wage models is the observation that higher absolute or relative company wages have positive production effects. This happens e.g. via higher effort performance at work, lower turnover rates, and the unemployment discipline effect (see Akerlof and Yellen 1990). In this context we use the generic efficiency wage model given by Solow (1979), where the labour input in production function is augmented with effort function,  $f(e(w)L)$ .

Assume that the firm has a CES production function with two distinct labour inputs  $L_1$  and  $L_2$  augmented by concave, positive and continuous effort functions  $e^1(w_1)$  and  $e^2(w_2)$ :

$$(2.1) \quad Q = \gamma[\tau h(\mathbf{x})^{-\rho} + \delta_1[e^1(w_1)L_1]^{-\rho} + \delta_2[e^2(w_2)L_2]^{-\rho}]^{-\nu/\rho}$$

where  $\tau = (1 - \delta_1 - \delta_2)$ ,  $h(\mathbf{x})$  represents some function of a vector of (quasi-fixed) non-labour inputs,  $\nu$  gives the degree of homogeneity (non-constancy of returns),  $\gamma$  is a scale parameter which can be used to denote technical efficiency, and  $\delta_i$ ,  $i=1,2$ , are the distribution parameters of inputs. The substitution parameter  $\rho$  is equal to  $(1 - \sigma) / \sigma$ , where  $\rho$  is elasticity of substitution.

Differentiating with respect to  $L_1, L_2, w_1$ , and  $w_2$ , and using marginal production conditions, produce after some manipulations (see e.g. Wallis 1979),

$$(2.2) \quad A_i \frac{Q^{1+\rho/\nu}}{[e^i(w_i)]^\rho} L_i^{-1-\rho} = w_i / p ,$$

$$(2.3) \quad A_i \frac{Q^{1+\rho/\nu}}{[e^i(w_i)]^\rho} (L_i^{-1-\rho}) \frac{\partial e^i / \partial w_i}{e^i(w_i)} = 1,$$

with  $i=1$  and  $2$ , and  $A_i = \delta_i \nu \gamma^{-\nu/\rho}$ . Log-linearizing (2.2) and (2.3) gives, for  $i=1$  and  $2$ ,

$$(2.4) \quad \ln L_i = c_i - \sigma \ln(w_i / p) - (1 - \sigma) \ln e^i(w_i) + \sigma(1 + \rho / \nu) \ln Q$$

$$(2.5) \quad \ln(\partial e^i / \partial w_i) - 1 / \sigma \ln e^i(w_i) = c_i + 1 / \sigma \ln L_i - (1 + \rho / \nu) \ln Q$$

$$\Leftrightarrow H_i(w_i) = c_i + 1 / \sigma \ln L_i - (1 + \rho / \nu) \ln Q$$

Both derived input specifications are non-linear. Equation (2.4) is partially linear, but equation (2.5) consists of an unknown complicated solution for  $w_i$ . Under the assumption that monotone  $H_i^{-1} = F_i$  exists, the following empirical specifications for the data ( $j=1, \dots, n$ ,  $i=1, 2$ ) is proposed:

$$(2.6) \quad \ln L_{ij} = \beta_{0i} + \beta_{1i} \ln(w_{ij} / p) + \beta_{2i} \ln Q + g_i(w_{ij}) + \varepsilon_{ij}^1$$

$$(2.7) \quad w_{ij} = F_i(\ln L_{ij}, \ln Q_i) + \varepsilon_{ij}^2$$

where  $\varepsilon_{ij}^1$  and  $\varepsilon_{ij}^2$  are *NID* random errors, and  $g_i(w_{ij})$  is some non-linear function of  $w_{ij}$ .  $F_i(\cdot)$  is an unknown non-linear function.

No assumption concerning the specific parametric form of  $e^i(w_i)$  is made. It is only assumed that  $e^i(w_i)$  is concave, continuous and at least twice differentiable in  $w_i$ . It will be estimated non-parametrically. Note that  $e^i(w_i) = \exp[g_i(w_i)/(\sigma-1)] > 0$  for all finite  $g_i(w_i)/(\sigma-1)$ , i.e.  $\sigma \neq 1$ . The economic interpretation of this result is interesting. If the elasticity of substitution between inputs is unity ( $\sigma = 1$ , Cobb-Douglas case), then the effort function  $e^i(w_i)$  is not identified (see also Eq. 2.4). Thus the CES approach is required for the identification of effort function.

The starting point is the hypothesis that  $e(w_i) \approx 1$ , since efficiency wages are not expected to have a big role in labour demand. This assumption is tested using specifications tests. The CES production function separable in the two types of labour is rather restrictive since it implies equal elasticity of substitution. However, the obtained estimates for elasticity of substitution below are same magnitude for different labour inputs indicating that the CES assumption is reasonable.

### 3. Semi-parametric and nonparametric regression models

Model (2.6) has the following form (the subscript  $i$  is left out in order to simplify the notation):

$$(3.1) \quad y_j = \mathbf{x}_j^T \boldsymbol{\beta} + g(w_j) + \varepsilon_j,$$

where for each observation  $y_j = \ln L_j$  there is four explanatory variables: a vector  $\mathbf{x}_j^T = (1 \ \ln(w/p)_j \ \ln Q_j)$  and scalar  $w_j$ .  $\boldsymbol{\beta}$ , a three dimensional vector of regression coefficients, and  $g(w_j)$ , a smooth curve, are to be estimated.  $\varepsilon_j$  is the *NID* error term.

Nonparametric regression techniques are used to model complex relationships between  $\{y_j\}_{j=1}^N$  and  $\{w_j\}_{j=1}^N$  without restricting  $g$  to parametric forms. When  $w \in [a, b]$ ,  $g \in W_2$ , where  $W_2 = \{g : g, g'$  are continuous, and  $\int_a^b [g''(w)]^2 dw < \infty\}$ , it is well known that the solution  $g_\lambda$  to the problem with fixed  $\beta$

$$(3.2) \quad \min_{g \in W_2} S(\beta, g) = \min_{g \in W_2} \left\{ \sum_{j=1}^N [y_j - \mathbf{x}_j^T \beta - g(w_j)]^2 + \lambda \int_a^b [g''(w)]^2 dw \right\}$$

is a cubic polynomial spline that smooths the data. Note that stated properties of  $g(w)$  ensure that a valid relationship will exist between it and the effort function  $e(w_j)$  defined in Section 2.1.  $\lambda > 0$  is a smoothing parameter that controls the trade-off between the infidelity to the data, as measured by the residual sum of squares, and to the roughness of  $g(w)$ , as measured by the  $L_2$ -norm of the second derivative of  $g(w)$  (see Craven and Wahba 1979, Yatchew 1998). For any given value of the smoothing parameter  $\lambda$ , minimizing  $S(\beta, g)$  in 3.2. will give the best compromise between smoothness and goodness-of-fit. When  $\lambda \rightarrow \infty$ ,  $g(w)$  is linear, and  $\lambda = 0$  produces a full fit. Thus, much depends on the value of the smoothing parameter  $\lambda$ .

The estimation of model (3.2) is directly related to LS methods if the integral part of the model can be expressed as some linear structure. This is precisely what the smoothing splines approach does (Engle et al. 1986, Green and Silverman 1994, Chapter 2):

$$(3.3) \quad (\mathbf{y} - \mathbf{X}\beta - \mathbf{g})^T (\mathbf{y} - \mathbf{X}\beta - \mathbf{g}) + \lambda \mathbf{g}^T \mathbf{K} \mathbf{g}$$

where  $\mathbf{K}$  is a positive semidefinite weight matrix of rank  $n-2$ . This can be re-written as a pair of simultaneous matrix equations after minimizing in respect to  $\beta$  and  $\mathbf{g}$ :

$$(3.4a) \quad \mathbf{X}^T \mathbf{X} \beta = \mathbf{X}^T (\mathbf{y} - \mathbf{g})$$

$$(3.4b) \quad (\mathbf{I} + \lambda \mathbf{K}) \mathbf{g} = (\mathbf{y} - \mathbf{X} \beta)$$

A *backfitting* algorithm gives estimates for  $\boldsymbol{\beta}$  and  $\mathbf{g}$  respectively converging to penalized least-square estimate,  $\hat{\boldsymbol{\beta}}_{PLS}$ , and smooth curve estimate,  $\hat{\mathbf{g}}$  (for more details, see Appendix 1).

It can be shown that Eqs. (3.4) give a consistent estimator of  $\mathbf{g}$  and  $T^{1/2}$  consistent normal estimator for  $\boldsymbol{\beta}$  (for more details, see Yatchew 1998). Different methods are proposed for choosing  $\lambda$ . In the following, the *cross-validation (CV) score* with  $RES_j = (y_j - \mathbf{x}_j^T \hat{\boldsymbol{\beta}}_{PLS} - \hat{\mathbf{g}}(w_j))$  is used

$$(3.5) \quad CV(\lambda) = N^{-1} \sum_{j=1}^N \left( \frac{RES_j}{1 - A_{jj}(\lambda)} \right)^2,$$

where  $A_{jj}$  is the diagonal elements of hat matrix  $\mathbf{A}$  (see Appendix 1). The CV method is a "leave-one-out" approach, where  $\hat{\mathbf{g}}$  is obtained by omitting the  $t^{th}$  observation. Thus the estimate of  $\mathbf{g}$  at each point  $w_j$  is obtained by estimating the regression using all other observations, then predicting the value of  $\mathbf{g}$  at omitted observation. The target is to find a value of  $\lambda$  that minimizes CV.

A natural approach in this context is to test the significance of the non-parametric nonlinear component  $\mathbf{g}(w)$  of the model. The  $H_0$  alternative is the pure parametric model, i.e.  $\mathbf{g}(w)$  does not add anything structurally. The tests derived by Hong and White (1995) and Hastie and Tibshirani (1990) that are related to the spline smoothing approach are applied here. Both tests assume that the smoothing parameter  $\lambda$  is optimally determined, i.e.  $MinCV(\lambda)$  exists.

According to the *M-test* of Hong and White (1995),

$$(3.8) \quad M = (N\hat{m}_N / \hat{\sigma}_\varepsilon^2 - p_N) / (2p_N)^{1/2},$$



where  $\hat{m}_N = N^{-1} \sum_{j=1}^N \hat{v}_j \hat{\epsilon}_j$  with  $\hat{v}_j = (y_j - \mathbf{x}_j^T \hat{\boldsymbol{\beta}}_{OLS})$  and  $\hat{\epsilon}_j = (y_j - \mathbf{x}_j^T \hat{\boldsymbol{\beta}}_{PLS} - \hat{g}_j)$ .  $p_N$  is a dimension correction value.  $\hat{\sigma}_\epsilon^2$  is the estimate for the residual variance under the null model. Hong and White suggest to use the closest integer of  $\ln N$  for  $p_N$ . In our case  $\ln(162)$  gives  $p_N = 5$ .

*F* test of Hastie and Tibshirani (1990) has the following form

$$(3.9) \quad F = \frac{(\sum_{j=1}^N \hat{v}_j^2 - \sum_{j=1}^N \hat{\epsilon}_j^2)(N - df_2)}{\sum_{j=1}^N \hat{\epsilon}_j^2 (df_2 - df_1)} \stackrel{\text{approx}}{\sim} F(df_2 - df_1, N - df_2) ,$$

where  $df_2$  represents the degrees of freedom of partial spline model ( $tr(2\mathbf{A} - \mathbf{A}\mathbf{A}^T)$ ) and,  $df_1$  is the degrees of freedom of OLS model without the non-linear part. Note that big a value of  $df_2$  means a very jagged  $\mathbf{g}$  curve.

The M test is asymptotically normally distributed. However, it is not supposed to perform optimally in small samples, like any procedure based on nonparametric regression. The approximation for F test given by Hastie and Tibshirani is not necessarily a good one in all cases. They suggest to use different moment corrections to obtain more reliable approximation to F-distribution. However, they are difficult to compute. Thus, the test distributions in this contexts are derived by bootstrapping the model residuals under the null alternative with *10,000* replicates and estimating semiparametric model (for more details, see Section 4.2).

Different estimators for residual variance  $\sigma_\epsilon^2$  are proposed in the literature (see Green and Silverman 1994, Chapter 3.4). One alternative is

$$(3.10) \quad \hat{\sigma}_\epsilon^2 = \frac{\sum_{j=1}^N RES_j^2}{\text{trace}(\mathbf{I} - \mathbf{A}(\lambda))}.$$

The pointwise standard error bands are computed using  $\pm 2[\hat{\sigma}_\epsilon^2 \text{diag}(\mathbf{A}\mathbf{A}^T)]^{1/2}$  under the assumption that the errors of model (3.1) are normal. Note that any residual-based 'diagnostic' tests are valid in

the partial linear model. This means that standard residual normality and heteroskedasticity tests, for example, can be carried out without modifications.

A closer look at the economic model in Eqs. (2.4) and (2.5) reveals that the wage rate  $w$  is an endogenous variable like labour input  $L$ . Output variable  $Q$  is assumed to be exogenous. At the disaggregated level output is demand constrained and imperfect competition is the natural market form. The consistent estimation of the wage equation demands non-linear instrumental variable estimation. However, the wage function is a non-linear unknown function of its arguments (see Eq. 2.5 and 2.7). Andrews (1991) and Newey (1990) showed that, in such a model, non-parametric methods give asymptotically efficient and normally distributed estimators. The case is also valid for the series estimator used here. The series estimator is

$$(3.11) \quad WS^{IV} = \mathbf{Z} \hat{\mu}, \quad \text{where}$$

$$\hat{\mu} = (\mathbf{Z}^T \mathbf{Z})^{-1} \mathbf{Z}^T \mathbf{y} \quad \text{with} \quad \mathbf{Z} = \mathbf{x}_1 \otimes \mathbf{x}_2 \dots \otimes \mathbf{x}_k,$$

where  $\mathbf{x}_r^T = (1 \ x_r \ x_r^2)$  for all  $r = 1, \dots, k$  exogenous instrument variables in the system ( $\otimes$  stands for the Kronecker product and  $i$  is a unit variable).  $\mathbf{y}$  is  $WS$ , vector of the observed wage rates. This means that  $\mathbf{Z}$  is a matrix that includes all possible combinations of 1st and 2nd degree terms of  $\mathbf{x}_r$ 's.

Note that the nonparametrically estimated  $WS^{IV}$  is used in two places to estimate the semiparametric model. First,  $WS^{IV}$  is an instrument for the knot values in function  $\mathbf{g}$ . Secondly, it is one of the instruments for estimating parameter  $\beta_1$  for the linear wage variable  $\ln(w/p)$  in the labour demand model (2.6). This prevents endogeneity bias in semiparametric estimations of  $\beta_1$  and  $\mathbf{g}$ .

## 4. The results

### 4.1. Data

Officially published cross-sectional data from Finnish industry for the year 1989 (with lowest digit levels) was used (Industrial Statistics, Central Bureau of Statistics in Finland). The sample consisted of 162 observations. The following variables were used for all different industries for  $i = 1$  and  $2$ : wage rate ( $WS$ ), number of hired persons ( $L =$  employment) and output ( $Q$ ). It was not possible to get separate data for output price,  $p$ , for the different industries. This means that no industry-specific price effects were allowed for in the estimations. They were buried in the nominal variables  $WS$  and  $Q$ . Note that this question is not relevant for the effort function estimation as workers' effort decisions are sensitive to *real consumption* wages. This means that all the wage observations should be deflated with same value of consumer price index. However, this type of scaling would not change the estimates for effort function since we are working with cross-section data from year 1989. Any other relevant variables obtained from the databases were used as instruments in IV estimation. Appendix 2. gives a more detailed description of the variables and the data set.

### 4.2. Model estimations

The industry statistics are quite uninformative about industry-specific factors that have an impact on labour demand. This was controlled as follows. Eleven dummy variables were constructed corresponding to the separate unions acting on different non-overlapping industries. At the same time, the dummies corresponded quite closely to the different aggregate industry output categories (food, clothing, printing, metals, etc.). Thus, aggregate industry-specific union and product effects were filtered away through OLS regression on  $\ln L_i$  with dummy variables  $D1, D2, \dots, D11$ . The fits of these models,  $\ln L_i^{DD}$ , are used below as a second endogenous variable. The estimated model is now

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1) All the calculations below were programmed with GAUSS -program. (The code is obtained from the author by request, see also Linden 1999).

$$(4.1a) \quad \ln L_{ij}^{DD} = \beta_{1i} \ln WS_{ij} + \beta_{2i} \ln Q_j + g^i(WS_{ij}) + \psi_{ij}^1$$

$$(4.1b) \quad WS_{ij} = F(\ln L_{ij}, \ln Q_j) + \psi_{ij}^2.$$

$\beta_{1i}$  in equation (4.1a) is estimated with *IV* method and (4.1b) is estimated with *IV* series estimator with  $\mathbf{Z}$  as defined in (3.11) containing also some auxiliary variables obtained from the data base (see Appendix 2). The partial spline method is also used to estimate  $g(WS_{ij})$  with  $WS_{ij}^{IV}$ . The following approach was used to build the knot values in  $\mathbf{g}_i$ : the values  $WS_{ij}^{IV}$  in the order of smallest to largest (according to which the data is sorted).

Before the final estimations were made the value of the smoothing parameters was determined. A search procedure among values  $\lambda = 0.001(1.1)^k, k=1, \dots, 200$  was run in order to find the  $\lambda_{opt}$  value that gave (local) minimum values for CV score. The unique values were  $\lambda_1 = 45$  (skilled workers) and  $\lambda_2 = 90$  (unskilled workers).

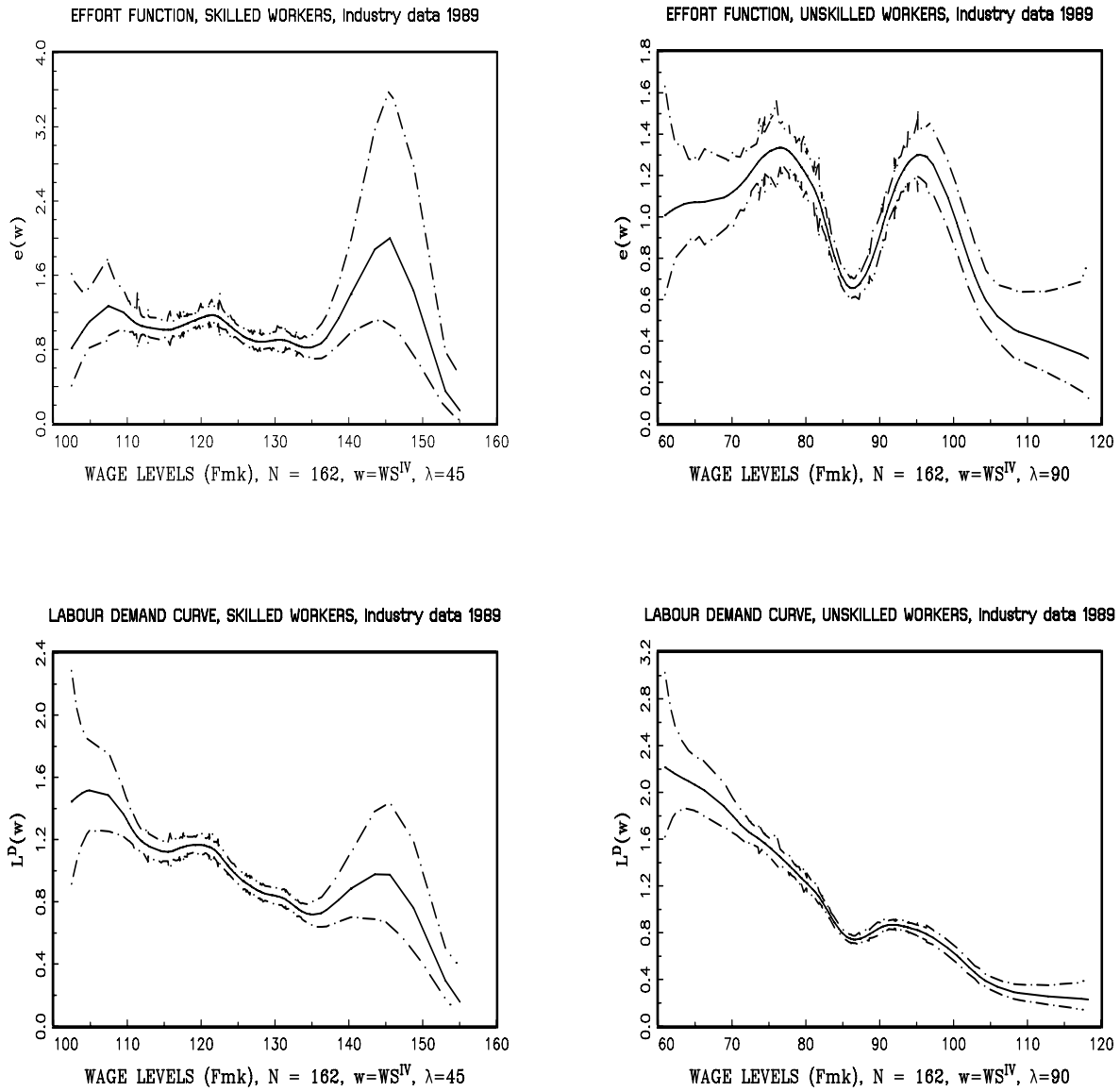
The curves in Figure 1. give the effort functions with a 95% -confidence intervals for skilled and unskilled workers. The  $e^i(w_i)$  functions are equal to  $\exp[\hat{\mathbf{g}}_i / (|\hat{\beta}_{1i}| - 1)]$ . The solution for skilled workers is imprecise at both ends because only few separate observations can be used for the end smoothing. When the four largest  $WS_{ij}^{IV}$  observations were excluded from the sample, the minimum for  $\lambda_1$  was 2500. This value produced a very flat unjagged effort function estimate which specification tests rejected. The curves on the lower part represent the plots of whole wage functions, i.e. the semiparametric estimates of the wage function

$$(4.2) \quad \ln L_{ij}^{DD} = c_1 \ln Q_i + G_i(WS_{ij}^{IV}) + \psi_{ij}^1.$$

The non-parametric curve estimates  $G_i(WS_{ij}^{IV})$  give at same time the total wage effects on labour demand, i.e. there is no distinction between the linear and non-linear parts. This is an estimate for

the labour demand curve as a function of wages, i.e.  $L_i^D(w_i)$ . The nonlinear parts are easily seen in the  $L_i^D(w_i)$  curves (for similar results with consumer gasoline demand, see Hausman and Newey 1995).

**FIGURE 1. EFFORT AND LABOUR DEMAND FUNCTION ESTIMATES WITH 95% CONFIDENCE INTERVALS FOR SKILLED AND UNSKILLED INDUSTRIAL WORKERS, 1989**



The estimate for unskilled workers is steeper and more jagged than the  $L^D(w)$  estimate for skilled workers if no attention is paid to the last six observations on the latter curve. Evidently, the non-linearities can be identified with the estimated effort functions. Note that the same  $\lambda_{opt}$  values as above may be used for  $e(w)$  estimations, because the spline smoothing is not affected by the presence of the linear parts of the labour demand equation.

Table 1. gives the parametric part of the semiparametric estimation. The specification tests for the semiparametric parts of the models gave interesting results. The small sample test distributions were derived with bootstrapping (10,000 replicates) the model residuals under the null hypothesis that  $\mathbf{g}_i = 0$ , and by estimating model (4.1a). In practice, the bootstrap test distributions were derived in the following way. New  $y_j^*$  values were calculated with model  $y_j^* = \mathbf{x}_j^T \hat{\beta}_{IV}^* + v_j^*$ , where  $v_j^*$  is the re-sampled residuals with a replacement. The bootstrap  $\hat{\varepsilon}_j^*$  and  $\hat{v}_j^*$  residuals were obtained with estimating the IV and the spline models with  $y_j^*$ , i.e.  $\hat{v}_j^* = y_j^* - \mathbf{x}_j^T \hat{\beta}_{IV}^*$  and  $\hat{\varepsilon}_j^* = y_j^* - \mathbf{x}_j^T \hat{\beta}_{IVPLS}^* - \hat{g}_j^*$ . Some bias correction methods suggested by Davison and Hinkley (1997) did not alter the results.

The bootstrapped p-values for the approximate F-test suggested by Hastie and Tibshirani show that estimated effort curve is significant at a 10% level for unskilled workers. A 5% significance level is obtained using Hong and White's test. However, both tests reject the skilled workers' effort function. The wage elasticities are quite close to each other, implying that the CES assumption used here may not be too restrictive. The t-tests for the equality of estimates of elasticity of substitution,  $\hat{\beta}_{1,skilled} = \hat{\beta}_{1,unskilled}$ , confirm this. The test value was below 1.2. Wage elasticities are relative large,  $-1.99$  for skilled workers and  $-2.17$  for unskilled. The evident reason is the large absolute value of elasticity of substitution between inputs. The 1987-1989 boom in Finnish industry led to an increase in labour demand. Other inputs were extensively substituted for labour.

The estimated effort function estimates may be interpreted as follows. The efficiency wages seem to play a role only for unskilled workers. The estimated effort function for unskilled workers has two different peaks and a notable downward trend. This means that effort performance is highest at

**TABLE 1. Semiparametric instrumental variable estimation of labour demand equation**

$$\ln L_i^{DD} = \beta_{1i} + \ln WS_{ij} + \beta_{2i} \ln Q_j + g^i(WS_{ij}) + \mu_{ij}^1$$

	SKILLED	UNSKILLED
lnWS	-2.186** (0.315)	-2.561** (0.327)
lnQ	0.769** (0.042)	0.831** (0.042)
Wage elasticity: $(1 - S_{Li})\hat{\beta}_{1i}$	-1.995	-2.171

Specification tests (p-values calculated using bootstrapping  $H_0$  residuals)

	(Hong & White 1995)	
M-test	3.164	5.912 **
(p-value)	(0.311)	(0.026)
	(Hastie & Tibshirani 1991)	
F-test	0.764	1.319*
(p-value)	(0.482)	(0.091)

Diagnostics

$R^2$	0.676	0.715
SE	0.692	0.603
$\chi_N^2(2)$	2.391	2.610
$\chi_H^2(1)$	1.272	1.584

\*\* ) significant at 5% level \* ) significant at 10% level

Heteroskedasticity consistent standard errors in parenthesis

$\chi_N^2(2)$ : A test for normality (Jarque-Bera 1980).

$\chi_H^2(1)$ : A test for heteroskedasticity (Koenker 1981).

the lowest wage levels, but that there is another local maximum at higher wage level too. Thus, among unskilled workers, two wage category classes exist which have distinct effort function regimes. The result may reflect a case where unskilled workers with lowest wages have the biggest threat of dismissal (the 'first in, first out' argument). This is the unemployment threat effect of efficiency wages. Thus, the unskilled workers with higher wages are the only group for which the Solow type of efficiency wage model is valid.

Another explanation for the observed effort function is the fact that unskilled workers in Finnish industry are typically young or female workers who are less likely to be unionized compared to skilled (male) workers. Unskilled jobs are often temporary and tasks are unspecified. A union membership is based on specific task and professional skill requirements performed in permanent jobs. The wages are not set at firm level but on the centralized level between unions and industry.

The rejection of the effort function for skilled workers implies that if effort extraction takes places, it does so with methods other than efficiency wages. Alternatively, any need for effort management among skilled workers can be redundant because their tasks and jobs are highly productive in themselves. Generally one may expect that firms have greater incentives to pay wage premia to skilled workers in order to keep the cost of turnover low. Thus, wage premia or drift may exist even if an effort function is not found.

## **5. Conclusions**

A direct method was introduced to estimate effort functions  $e(w)$  for unskilled and skilled workers. Partially linear models including non-parametric functions of wages were estimated with PLS method. The data consisted of cross-sectional industry observations from the year 1989 in Finland. The effort function estimate based on partial smoothing splines was not rejected for unskilled workers by the specification tests used. However, the shape of the function demands a more detailed analysis that pays attention to the different wage categories among unskilled workers. The effort function for skilled workers was rejected by the data. The semiparametric labour demand estimates showed clearly non-linear curvatures identified with the effort functions.



## APPENDIX 1. Partial smoothing spline estimation

A function  $\mathbf{g}$  is defined on intervals  $(t_i, t_{i-1}) \in [a, b]$  as a natural cubic spline if i)  $\mathbf{g}$  is a cubic polynomial in each interval  $t_1 < t_2 < \dots < t_n$ , ii) the polynomial pieces fit together at points  $t_i$  in such a way that  $\mathbf{g}$  itself and at least the first two derivatives are continuous at each knot  $t_i$ , and, iii)  $\mathbf{g}'$  and  $\mathbf{g}''$  are zero at the end points  $a$  and  $b$ . Thus (see Green and Silverman, 1994, Chapter 2):

$$\begin{aligned}
 (A1.1) \quad \int_a^b \mathbf{g}''(t)^2 dt &= \sum_{i=1}^{n-1} \gamma_i \left[ \frac{\mathbf{g}_{i+1} - \mathbf{g}_i}{h_i} - \frac{\mathbf{g}_i - \mathbf{g}_{i-1}}{h_{i-1}} \right] \\
 &= \boldsymbol{\gamma}^T \mathbf{Q}^T \mathbf{g} = \boldsymbol{\gamma}^T \mathbf{R} \boldsymbol{\gamma} \\
 &= \mathbf{g}^T \mathbf{Q} \mathbf{R}^{-1} \mathbf{Q}^T \mathbf{g} = \mathbf{g}^T \mathbf{K} \mathbf{g}
 \end{aligned}$$

$$\text{iff } \mathbf{Q}^T \mathbf{g} = \mathbf{R} \boldsymbol{\gamma},$$

where  $\mathbf{g}^T$  is vector  $(g_1 \dots g_n)$ , and  $\boldsymbol{\gamma}^T = (\gamma_2 \dots \gamma_{n-1})$ , a vector of  $\mathbf{g}''(t_i)$  for  $i = 2, \dots, n-1$ .  $\mathbf{Q}$  and  $\mathbf{R}$  are band matrices defined as weighted inversions of  $h_i = t_{1+i} - t_i$  for  $i = 1, \dots, n-1$ .

This construction enables the penalized least square to be written as

$$(A1.2) \quad S(\boldsymbol{\beta}, \mathbf{g}) = (\mathbf{y} - \mathbf{X}\boldsymbol{\beta} - \mathbf{g})^T (\mathbf{y} - \mathbf{X}\boldsymbol{\beta} - \mathbf{g}) + \lambda \mathbf{g}^T \mathbf{K} \mathbf{g}$$

Now the fit of the model and the hat matrix are  $(\mathbf{S} = (\mathbf{I} + \lambda \mathbf{K})^{-1})$

$$(A1.3) \quad \hat{\mathbf{y}} = \mathbf{A} \mathbf{y} = \mathbf{X} \hat{\boldsymbol{\beta}} + \hat{\mathbf{g}}, \text{ where}$$

$$(A1.4) \quad \mathbf{A} = \mathbf{S} + (\mathbf{I} - \mathbf{S}) \mathbf{X} [\mathbf{X}^T (\mathbf{I} - \mathbf{S}) \mathbf{X}]^{-1} \mathbf{X}^T (\mathbf{I} - \mathbf{S})$$

$$(A1.5) \quad \hat{\boldsymbol{\beta}} = [\mathbf{X}^T (\mathbf{I} - \mathbf{S}) \mathbf{X}]^{-1} \mathbf{X}^T (\mathbf{I} - \mathbf{S}) \mathbf{y}$$

$$(A1.6) \quad \hat{\mathbf{g}} = \mathbf{S} (\mathbf{y} - \mathbf{X} \hat{\boldsymbol{\beta}}).$$

A semiparametric IV estimation is conducted with

$$(A1.7) \quad \hat{\boldsymbol{\beta}}_{IV} = (\mathbf{Z}^T \mathbf{X})^{-1} \mathbf{Z}^T \mathbf{y},$$

where  $\mathbf{Z} = \mathbf{X}_{IV}^T (\mathbf{I} - \mathbf{S}_{IV})$ .  $\mathbf{X}_{IV}^T$  is a set of instrumental variables and  $\mathbf{S}_{IV}$  is calculated using  $WS^{IV}$ .

## APPENDIX 2. The data and variables used

$i = 1$  (skilled worker),  $i = 2$  (unskilled worker).

Skilled workers in a firm are persons whose jobs and tasks need some special training or human capital to be performed successfully. Unskilled workers' tasks need no general training or schooling. The workers may have some firm-specific training, but in general they are perform manual, non-professional (blue-collar) jobs.

$L_{ij}$  = employment in industry  $j$ .

$Q_j$  = output in industry  $j$ .

$WS_{ij}$  = (wage sum + social security payments)/ $(L_{ij}H_{ij})^{1/2}$  ,

where  $H_{ij}$  = hours worked in industry  $j$ .

In addition, the statistical resources used allowed for the following variables: Export rate ( $EXPR_i$  = Exports/Output), gross investment ( $INV_i$ ), productivity ( $PR_{ij} = \ln(Q_i/L_{ij})$ ), and measure of concentration ( $TPR_i$  = number of firms in industry/ employment),  $j = 1, \dots, 162$ . However, only  $EXPR_{ij}$  and  $TPR_i$  turned out to be significant in the preliminary estimations of labour demand equations. Thus only these two variables were used as auxiliary *IV* variables in the estimations.

The nonparametric instrumental series estimate,  $WS^{IV} = \mathbf{Z}(\mathbf{Z}^T \mathbf{Z})^{-1} \mathbf{Z}^T \mathbf{y}$ , was calculated using the instrument vector

$\mathbf{Z} = \mathbf{x}_1 \otimes \mathbf{x}_2 \otimes \mathbf{x}_3$ , where

$$\mathbf{x}_1^T = (i \quad \ln Q \quad (\ln Q)^2)$$

$$\mathbf{x}_2^T = (i \quad \ln TRP \quad (\ln TRP)^2)$$

$$\mathbf{x}_3^T = (i \quad \ln EXPR \quad (\ln EXPR)^2) \quad \text{and}$$

$$\mathbf{y} = WS$$

## REFERENCES

- Ackum Agell, S.(1994) "Swedish evidence on the efficiency wage hypothesis", *Labour Economics*, 1, 129-150.
- Akerlof, G.A. and Yellen, J.L. (1986) "Introduction" in Akerlof, G.A. and Yellen, J.L. (eds.) *Efficiency wage models and the labor market*, Cambridge Univ. Press, 1-20.
- Andrews, D.W.K. (1991) "Asymptotic normality of series estimators for nonparametric and semiparametric regression models", *Econometrica*, 59,307-345.
- Craven, P. and Wahba, G. (1979) "Smoothing noisy data with a spline functions", *Numerische Matematik*, 24, 375-382.
- Davison, A.C. and Hinkley, D.V. (1997) *Bootstrap Methods and their Application*, Cambridge University Press, Cambridge.
- Engle,R.F.,Granger,C.W.J.,Rice,J.A. and Weiss, A. (1986) "Semiparametric estimates of the relation between weather and electricity sales", *Journal American Statistical Association*, 81, 310-20.
- Fuess, S.M. and Millea, M. (2002) "Do employers pay efficiency wages? Evidence from Japan", *Journal of Labor Reserch*, 23, 279-292.
- Green, P.J. and Silverman, B.W. (1994) *Nonparametric Regression and Generalized Linear Models*, Chapman and Hall, London.
- Goldsmith, A.H., Veun, J.R. and Darity, W. (2000) "Working hard for the money? Efficiency Wages and Worker Effort", *Journal of Economic Psychology*, 21, 351-385.
- Hastie, T.J. and Tibshirani, R.J. (1991) *Generalized Additive Models*, Chapman and Hall, London.
- Hausman, J. A. and Newey, W.K. (1995) "Nonparametric estimations of exact consumers surplus and deadweight loss", *Econometrica*, 63, 1445-1476.
- Jarque, C.M. and Bera, A.K. (1980) "Efficient tests for normality, homoskedasticity and serie independence of regression residuals", *Economic Letters*, 6, 255-229.
- Hong, Y. and White, H. (1995) "Consistent specifications testing via nonparametric series regression", *Economterica*, 63, 1133-1159.
- Kitazawa, Y. and Ohta, M. (2002) "Testing the shrinking version of the efficiency wage model in the Japanese electric-machinery firms: A panel data approach", *Applied Economics Letters*, 9, 335-338.

- Koenker, R. (1981) "A note on studentizing a test for heteroskedasticity", *Journal of Econometrics*, 17, 107-112.
- Konings, J. and Walsh, P.P. (1993) "Evidence of efficiency wage payments in UK firm level panel data", *Economic Journal*, 104, 542-555.
- Krueger, A.B. and Summers, L.H. (1988) "Efficiency wages and the inter-industry wage structure", *Econometrica*, 56, 259-293.
- Kugler, A.D. (2003) "Employee referrals and efficiency wages", *Labour Economics*, 10, 531-556.
- Kumbhakar, S.J. (1996) "A farm-level study of labor use and efficiency wages in Indian agriculture", *Journal of Econometrics*, 72, 177-195.
- Machin, S. and Manning, A. (1992) "Testing dynamic models of worker effort", *Journal of Labor Economics*, 10, 288-305.
- Linden, M (1999) "Estimating Effort Function with semiparametric Model", *Computational Statistics*, 14, 501-513.
- Newey, K.N. (1990) "Efficient instrumental variables estimation of nonlinear models", *Econometrica*, 58, 809-837.
- Solow, R. (1979) "Another possible source of wage stickness", *Journal of Macroeconomics*, 1, 79-82.
- Wadhvani, S. and Wall, M. (1991) "A direct test of the efficiency wage model using UK micro-data" *Oxford Economic Papers*, 43, 529-548.
- Wallis, K.F (1985) *Topics in applied econometrics*. 2nd ed. Basil Blackwell, London.
- Yatchew, A. (1998) "Nonparametric regression techniques in economics", *Journal of Economic Literature*, 36, 669-721.